**Example 1:** Two small samples of solids are introduced into a constant environment oven and kept there for 1h. Under these conditions, the 4mm particles are 58% converted and 2 mm particles are 87.5% converted.

(a) find the rate controlling mechanism for these cases

(b) Find the time needed for complete conversion of 1 mm particles in the same oven.

**Solution:**

<table>
<thead>
<tr>
<th>R (mm)</th>
<th>Time (h)</th>
<th>X_B</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>0.875</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.58</td>
</tr>
</tbody>
</table>

If diffusion through gas film is rate limiting, 

\[ t = \frac{\rho_B R}{3k_g C_{Ag}} \left( 1 - \left( \frac{r_c}{R} \right)^3 \right) \]

If diffusion through ash shell is rate limiting, 

\[ t = \frac{\rho_B R^2}{6D_e C_{Ag}} \left( 1 - 3 \left( \frac{r_c}{R} \right)^2 + 2 \left( \frac{r_c}{R} \right)^3 \right) \]

If surface reaction is rate limiting, 

\[ t = \frac{\rho_B R}{k_s C_{Ag}} \left( 1 - \frac{r_c}{R} \right) \]

The time is fixed, and the parameters \( k_s, k_g, D_e \) and \( \rho_B \) are constants. If diffusion through gas film is rate limiting then

\[ \frac{3k_g C_{Ag}}{\rho_B} = \frac{R}{t} \left( 1 - \left( \frac{r_c}{R} \right)^3 \right) = \text{constant1} \]

If diffusion through ash shell is rate limiting then,

\[ \frac{6D_e C_{Ag}}{\rho_B} = \frac{R^2}{t} \left( 1 - 3 \left( \frac{r_c}{R} \right)^2 + 2 \left( \frac{r_c}{R} \right)^3 \right) = \text{constant2} \]

If surface reaction is rate limiting, then

\[ \frac{k_s C_{Ag}}{\rho_B} = \frac{R}{t} \left( 1 - \frac{r_c}{R} \right) = \text{constant3} \]

<table>
<thead>
<tr>
<th>R (mm)</th>
<th>Time (h)</th>
<th>X_B</th>
<th>( \frac{r_c}{R} ) (^3)</th>
<th>( \frac{r_c}{R} )</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>0.875</td>
<td>0.125</td>
<td>0.5</td>
<td>1.75</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
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<td>1</td>
<td>0.58</td>
<td>0.42</td>
<td>0.75</td>
<td>2.32</td>
<td>2.44</td>
<td>1</td>
</tr>
</tbody>
</table>

A look at the above table indicates that the surface reaction is rate limiting and that

\[ \frac{k_s C_{Ag}}{\rho_B} = 1 \text{ mm h}^{-1} \]

Therefore, 1 mm particle will take \( \tau = \frac{\rho_B R}{k_s C_{Ag}} = 1 \text{ h mm}^{-1} \times 1\text{mm} = 1\text{h} \) for complete conversion.